# MODELING TRANSPORTATION AFFORDABILITY WITH THE CUMULATIVE DENSITY FUNCTION OF THE MATHEMATICAL BETA DISTRIBUTION

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## ABSTRACT

Transportation affordability refers to people's financial ability to access important goods and activities such as work, education, medical care, basic shopping and socializing. Making transportation more affordable can produce considerable socio-economic benefits by lowering the costs and boosting mobility for people that are more disadvantaged. More affordable transportation is equivalent to higher income.

There are many factors to consider when evaluating transportation affordability, including housing affordability; land use factors that affect accessibility; the quantity, quality and pricing of mobility options; and individuals' mobility needs and abilities. Traditional transportation planning hardly takes into account any transportation affordability considerations. Greater emphasis on this field would shed more light on affordability impacts and help policy makers to identify more affordable transportation solutions. However, to take transportation affordability into account there should be practical ways of evaluating it.

This paper investigates the concept of transportation affordability and suggests a metric for its measurement. The metric calculates affordability based on the tradeoffs that households make between transportation and housing costs. The transportation costs considered include car ownership, car use and public transport costs. The suggested approach can be applied to any spatial zone (e.g. neighborhood or other) to reflect the average expenditure that households are willing to make to satisfy their basic travel needs.


**Keywords:** Transportation affordability, household transportation costs, smart growth planning, public transport planning, non-automotive means, housing location efficiency, affordability metric.

## INTRODUCTION

Affordability, in principle, refers to people's ability to purchase important goods and services. In a similar vein, transportation affordability refers to people's financial ability to access important goods and activities such as medical care, basic shopping, education, work and socializing *(1)*.

Transportation inaffordability causes significant problems since it imposes financial burdens on lower income households and constrains people's opportunities. Some people might have to travel longer distances in poor-condition vehicles increasing the levels of stress and contributing to traffic hazards. Some others might prefer state support from moving to locations with more jobs but high living costs *(2)*. Of course, all households are not equally affected by inaffordability. Some may accept inferior housing in return for improved economic perspective and better living conditions. But when this group of people falls short of the demand for labor, businesses must increase salaries to attract new employees. This tends to increase production costs and decrease market competitiveness, which would have been (partially) avoided if cheaper housing and more affordable transport was available. It is apparent that considering transportation affordability in the design of transport policies and strategies is very important, particularly in remote or isolated areas *(3)* and in fast developing communities that rely primarily on low-medium income workers or wish to attract pensioners or students.

Many planning decisions influence the affordability of transportation. Usually transportation planning is focused on the needs of wealthy travelers but respond less well to the needs of the poor *(1)*. This is advocated by the large number of investments in car and freight transport, as opposed to initiatives to improve more affordable means such as public transport and non-automotive modes *(4)*. The above exacerbate the financial problems of low-income households because it increases the access costs to employment and education. Moreover, because motorized transport is resource-intensive, it contributes to environmental degradation and increases the dependence on imported fuels *(5)*.

There are several factors influencing transportation affordability and a variety of means to attain more affordable solutions, but some of them are currently overlooked by planning practices. Affordable transportation can be achieved by introducing smart growth planning and less car-dependent transportation options *(6)*. These strategies result in increasing the number of nearby destinations, making trips shorter and more accessible with non-automotive means and as such they contribute in congestion reduction, improved safety and health. Moreover, some transport affordability strategies are mere economic transfers, i.e. cost shifts rather than true cost reductions, which tend to be economically inefficient because they violate the principle that prices reflect marginal costs, and so encourage inefficient consumption. For example, driving is made more affordable by financing parking facilities within building budgets which reduces housing affordability, a portion of roadway costs are borne through general taxes rather than user fees, and automobile insurance is made affordable to higher-risk drivers by overcharging lower-risk drivers.

Apparently, the concept of transportation affordability is important for transport and spatial development planning, and as such it should not be overlooked in any relevant decision. To promote affordable transportation requires a robust framework that defines and measures transportation affordability appropriately. This paper suggests a framework for analyzing transportation affordability and reflecting it upon a quantitative scale. The estimation of households' transportation expenditures using the policy-related variables of residential/job density and center proximity, and the consideration of household characteristics may provide policy makers with additional tools for suggesting more affordable transportation solutions.

## REVIEW OF PREVIOUS RELATED WORK

Various methods can be found in the literature that can be used to evaluate transportation affordability. Most of them take into account the portion of household income devoted to transportation, housing affordability, the characteristics of available transportation options and the accessibility to employment and local shopping centers, for public transit users compared with car users.

Leigh et al *(7)* have developed a method for assessing transportation affordability based on the concept of mobility gap. The authors define the mobility gap "as the amount of additional transit service required for households without a motor vehicle to have a comparable level of mobility as vehicle owning households". The larger the mobility gap of a community the less affordable is transportation in that area.

The method considers a variety of factors when assessing the public transport needs of a community and the mobility gap between users with or without a car. These factors include car ownership, age (people between 10-21 and 65+ years seem to be more dependent on public transit than those between 21 and 65), income (lower-income people seem to use public transport more), and residential status (immigrant residents tend to rely more on transit than native residents). In the locations evaluated by the authors it was found that only about a third of transit needs were met.

An obvious problem with the Leigh et al. approach is that it fails to answer questions like: how much transit would be necessary to achieve auto level-of-service? Also additional work is needed to assess the cost of achieving such transit level-of-service (i.e., how much taxation would likely go up to finance it and the likely affordability impacts on other goods and services).

Another interesting approach to transportation affordability is the "transit-oriented development method". In a transit-oriented community, households' transportation costs tend to be reduced to the benefit of transportation affordability. Recent research *(8)*, *(9)*, *(10)*, suggests that the total amount spend in transportation by households tends to decrease with increased use of public transport and is, generally, lower in 'large rail cities' (as called in this research the areas with high quality transit systems). People in large rail cities spend less that 12.0% of their income to transportation, while in 'small rail cities' (cities with modest rail transit systems) spend about 15.8%, and in 'bus-only cities' (cities that lack rail transit) an average of 14.9%. International comparisons show similar patterns *(11)*. It should be noted that comparisons showing the shares of income spent in transportation might miss out on higher incomes associated with large cities. They might also miss out on people spending longer amounts of time traveling, due to congestion and to a higher share of people using public transportation. Another shortcoming of the method is related to its failure to consider possible means of transit financing and to assess the way they might affect the economic condition of the poor.

According to Litman *(12)* a significant number of cost-reduction policies, such as decreasing fuel taxes and subsidized tolls and parking, may result in increased car affordability but may also cause other costs to soar, for example, housing costs or taxes. If any of these indirect costs are incurred by people of lower income they may lead to a reduction in affordability. Likewise, transportation affordability may decline if under-priced car use results in more traffic congestion, accidents and environmental degradation, especially if these external impacts affect lower income people. To assess transportation affordability in this context, one must distinguish economic transfers (e.g. subsidies and environmental externalities) from real costs of resources and resource cost savings. There also some methodological implications to consider, for example, how to address a wide range of external or indirect costs and benefits which are not always very easy to compute.

The Center for Neighborhood Technology (CNT) and the Center for Transit Oriented Development (CTOD) have developed the Housing and Transportation Affordability Index *(13)*, which appears to be the most workable approach to affordability to date. The Index estimates true housing affordability by considering its locational value which is measured through transportation costs. According to this research car ownership, car use and public transport ridership are the main dependent variables in the household transport cost model.

The above literature review shows that existing approaches to the measurement of transportation affordability overlook the increased variation in transportation resources and costs across household groups and locational settings. To address this limitation, this paper proposes a Transportation Affordability Metric based on the tradeoffs that households make between transportation and housing costs which reflect also the location efficiency.

## BUILDING AN AFFORDABILITY METRIC

The Transportation Affordability Metric (TAM) builds on the analysis of the Housing and Transportation Affordability Index. Its added value regarding existing state of research lies with the fact that it uses a fully parameterized mathematical function which allows fine-tuning to the housing, transportation costs, household expenditures and other factors that effect affordability such as location and user characteristics. Before introducing the TAM, it will be useful to set out what are the key aspects to be taken into account for the overall development of the metric.

### Main aspects

*Transportation affordability analysis should consider housing and transportation costs together*

Households usually face tradeoffs between transport costs, housing costs and income. It is common to find lower-cost housing in remote locations with high transportation costs, which means no overall gain in affordability. According to Litman *(14)*, transport costs for middle income households in the US range from about 10% in urban areas to about 25% in less dense car-dependent locations. Miller, et al *(15)* found similar results in the Toronto region, estimating that a typical household would spend about €4.200 annually in additional motor vehicle costs if located in a suburban area. Using data from the Minneapolis-St. Paul region, Makarewicz, et al *(16)* found that low-cost housing in areas with good transportation services results to an increase of overall affordability *(13)*, *(17)*, *(18)*.

*Transportation affordability should consider a variety of transportation costs*

There are various specific costs that affect affordability, including: vehicle purchase costs and fees, road tolls and parking fees, public transport and taxi tariffs and gasoline prices. For example, an increase in vehicle registration and insurance fees might result in reduced transportation affordability. But tradeoffs also exist, e.g. a reduction in fuel prices may encourage a more sprawled, automobile-dependent urban pattern, resulting in no overall gain in affordability. For all these reasons it is important that any attempt to develop a Transportation Affordability Metric should take into account car ownership, car use and transit use costs and be based on total rather than unit costs.

*Transportation affordability should be evaluated relative to total expenditures*

Affordability analysis may follow different paths as definitions and perspectives vary. Analysis results, for example, may be different if costs are measured relative to income or expenditures, if state-aid is considered as income (many households have undeclared income, live in subsidized houses, receive non-income benefits) and if parking costs are included in housing costs or in transportation costs.

Transportation expenditures are regressive when measured relative to household incomes, but not relative to household expenditures *(19)*, *(20)*. For instance, retired people spend more than their current incomes (because they are living on savings) and have reduced mobility needs because they are aged or possibly disabled. Another reason for that is that incomes are usually higher in cities where transit is available. These factors help explain better why it was decided in this paper to evaluate affordability on the basis of households' total expenditures and not relatively to income, which is the common case. For a more detailed discussion on the advantages of using households' expenditures the reader can refer to Litman *(1)*.

In the following section it will be described how the total transportation cost is estimated for the needs of the TAM.

## Transportation Costs

To estimate transportation costs, this paper builds on the theory of the Location Efficient Mortgage - LEM *(21)*. The LEM uses vehicle-miles traveled for households in the Southern California, the Chicago region and the San Francisco bay area to produce fitting models assessing car ownership and car use, based on measures of residential density, public transport availability, and neighborhood friendliness for pedestrians or cyclists. The LEM yields a 'location efficient value' at a neighborhood level within these regions.

In the Transportation Affordability Metric, household transportation costs are estimated as three separate components: car ownership (*Co*), car use (*Cu*), and transit use (*Cp*) costs. These three components are the dependent variables of the model and are associated with six independent locational variables and two independent household variables (household expenditure and size). Together, these eight variables (table 1) represent the independent spatial and socio-demographic variables that are used for the estimation of household transportation costs (Equation 1).

$$TTC = [Co*Fo(Ve)*Go(Vh)]+[Cu*Fu(Ve)*Gu(Vh)]+[Cp*Fp(Ve)*Gp(Vh)] \quad (1)$$

Where TTC is the Total Transportation Cost, $C_x$ is a cost factor (e.g, Euros per km driven), and $F_x$ and $G_x$ are general functions representing the characteristics of the local environment (*Ve*) and the household income and size (*Vh*).

**TABLE 1 Variables used for estimating the households' transportation costs**

| Independent variable | Data Source | Purpose |
|---|---|---|
| Households per residential square-km | Census | Provides a measure of density, which influences car ownership and use |
| Population per total square-km | Census | Provides a measure of density, which influences car ownership and use |
| Zonal transit density* | Bus operators, local transit agencies | Provides a measure of transit accessibility |
| Distance to employment centers | Census | Distance to nearby jobs influences car ownership and car use |
| Job density: number of jobs per square-km | Census / Jobs and locations | Proximity to nearby employment center affects car ownership, car and transit use |
| Access to amenities | Census / Service jobs | Nearby services affects car ownership, car use and transit use |
| Household expenditure | Census | Influences car ownership and use |

| | | |
|---|---|---|
| | | |
| Household size | Census | Influences car ownership and use |
| **Dependent variable** | **Source** | **Use** |
| Car ownership (vehicles per household) | Modeled from independent household and local environment variables | To assess the number of cars owned by a household and the associated costs |
| Auto use (annual km driven per household) | Modeled using census data fitted to the independent variables | To assess the number of km driven by household's vehicles and the associated costs |
| Transit Rides per day | Modeled from independent household and local environment variables | To determine the number of transit rides per day per household |

* *Daily average number of buses or trains per hour, times the fraction of the zone within 400m of each bus stop (or 800m of each rail or ferry stop or station), summed for all transit routes in or near the zone.*

For a more detailed discussion on the significance of the above variables in the transport cost model the reader can refer to Holtzclaw et al *(21).* As shown in that study, all independent variables of the model correlate with all the dependent ones, but with different strength. The variables, for example, that correlate most strongly with car ownership and car use are the residential density variables[1] i.e. households/residential square-km, and population/residential square-km. Similarly, the variables that correlate more with transit ridership are transit and job density.

Bounded power fits ($y=A*[(x+B)/(X_{avg}+B)]^D$) give the strongest single-independent-variable correlations which means that they can be used as the basis for the development of the $F_x$ and $G_x$ functions. Of course, the differences between the calibration parameters $A$, B and $D$ reflect the zone-to-zone disparities in mobility patterns, level of accessibility, terrain layout, etc.

## Formulation of the Metric

As already mentioned the added value that the suggested TAM brings to existing research is the fully parameterized mathematical function which can be fine-tuned to key factors effecting affordability. From a mathematical perspective the Transportation Affordability Metric is a continuous, smooth function which varies with transportation cost, while satisfying a series of affordability properties such as:

1. Considers housing and transportation costs together
2. Decreases when transportation cost increases
3. Plunges steeply when housing cost increases
4. Yields zero when transportation cost reaches a threshold $C$ and one when it tends to zero.

In its general form the suggested TAM is shown in Equation 2 below.

$$A^{C}_{\kappa,\lambda}(T,H) := \begin{cases} B_{\kappa,\lambda}\big(1 - T/(C-H)\big) & \text{if } 0 \le T + H \le C \\ 0 & \text{otherwise} \end{cases} \qquad (2)$$

where, $B_{\kappa,\lambda}$ is the Beta function (see Equation 3), $T$ is the total transportation cost, $H$ is the housing cost and $C$ a positive constant reflecting households' decision heuristics.
The development of the Metric has followed a framework, comprising of three steps:

- Selection of mother-function and basic transformations

[1] Vehicles per household and km driven tend to decrease as residential density increases.

- Calibration of derived function
- Final configuration of the Metric

*Selection of mother-function and basic transformations*

A family of functions adhering properly to the required conditions is the well known Cumulative Density Function of the mathematical Beta Distribution *(22)*. This family has formed the basis for the development of the Transportation Affordability Metric, and underwent serious transformation in order to reflect the required properties.

Let us introduce the Beta Cumulative Density Function[2] (Equation 3).

$$B_{\kappa,\lambda}(z) := \frac{\int_0^z t^{\kappa-1}(1-t)^{\lambda-1}\,dt}{\int_0^1 t^{\kappa-1}(1-t)^{\lambda-1}\,dt} = \frac{\Gamma(\kappa+\lambda)}{\Gamma(\kappa)\Gamma(\lambda)}\int_0^z t^{\kappa-1}(1-t)^{\lambda-1}\,dt \qquad (3)$$

where, $\kappa$, $\lambda$, $z$ are complex numbers, and $\Gamma$ a special function known as Gamma or Factorial Function *(23)*.

Apparently $B_{\kappa,\lambda}(0)=0$, $B_{\kappa,\lambda}(1)=1$ and $B_{\kappa,\lambda}$ is strictly increasing in the unit interval [0,1] with values from 0 to 1. To adjust to the required boundary conditions (property 4) and make the Cumulative function strictly decreasing (property 2), the range of the function was mapped from the unit interval to the interval [0,*C*]. Its monotonicity was also inverted. The composite function that resulted for this transformation is shown below (Equation 4):

$$f^{C}_{\kappa,\lambda}(T) := \begin{cases} B_{\kappa,\lambda}\left(1-T/C\right) & \text{if } \; 0 \le T \le C \\ 0 & \text{otherwise} \end{cases} \qquad (4)$$

where, *C* is a positive constant less than 1 and *T* is the portion of total household expenditure devoted to transportation.

To simplify this function we applied term-by-term integration for integer values of $\kappa$ and $\lambda$, which resulted in $B_{\kappa,\lambda}(1-\mathrm{T}/C)$ taking the following form (Equation 5):

$$\begin{aligned} B_{\kappa,\lambda}\left(1-T/C\right) &= \frac{(\kappa+\lambda-1)!}{(\kappa-1)!(\lambda-1)!}\sum_{i=0}^{\lambda-1}\binom{\lambda-1}{i}\frac{(-1)^i}{(\kappa+i)}\left(\frac{C-T}{C}\right)^{\kappa+i} \\ &= \frac{(\kappa+\lambda-1)!}{(\kappa-1)!}\sum_{i=0}^{\lambda-1}\frac{(-1)^i}{(\lambda-1-i)!\,i!\,(\kappa+i)}\left(\frac{C-T}{C}\right)^{\kappa+i} \end{aligned} \qquad (5)$$

where $\binom{\lambda-1}{i}$ stands for the binomial coefficient sequence in the expanded form of $(1-\cdot)^{\lambda}$.

In the above, $f^{C}_{\kappa,\lambda}$ is a ($\kappa+\lambda$ -1) degree polynomial of *T* defined in [0,*C*], having *C* as root with multiplicity $\kappa$. Further, the polynomial $1-f^{C}_{\kappa,\lambda}(\cdot)$ has 0 as root with multiplicity $\lambda$.

Figure 1 (left-side) illustrates the different shapes of the $f^{C}_{\kappa,\lambda}$ function assuming $\kappa=\lambda=3$[3] and *C* ranging from 0.1 to 0.5 with step 0.1.

It can be seen that this family of functions satisfies the boundary conditions (property 4) and the requirement of decreasing monotonicity (property 2). It also satisfies property 3; that is providing for different decreasing rates as *C* varies.

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[2] Also known as the Regularized Incomplete Beta Function *(24)*.

[3] Here $f^{C}_{\kappa,\lambda}$ takes the polynomial form $f^{C}_{\kappa,\lambda} = (C-T)^3\left(6T^2+3CT-C^2\right)C^{-5}$

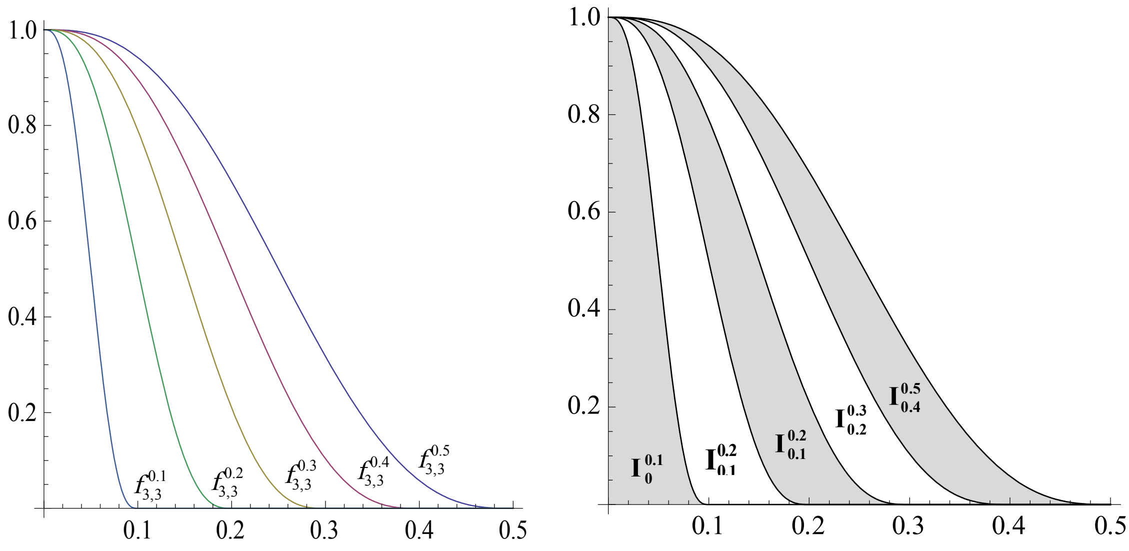


**FIGURE 1 Alternative shapes of the $f_{\kappa,\lambda}^{C}$ function**

*Calibration of derived function*

To calibrate $f_{\kappa,\lambda}^{C}$, the $\kappa$ and $\lambda$ parameters must be set. This requires a clear understanding of the relations between $T$, $C$, $\kappa$ and $\lambda$ and of how they affect the function.

To investigate the relations between the parameters we integrated Equation 4. Then by changing the order of integration[4] of the next double integral we take (Equation 6):

$$\int_0^C \int_0^{1-T/C} t^{\kappa-1}(1-t)^{\lambda-1} dt\, dT = \int_0^1 \int_0^{C(1-t)} t^{\kappa-1}(1-t)^{\lambda-1} dT\, dt = C\int_0^1 t^{\kappa-1}(1-t)^{\lambda}\, dt = C\frac{\Gamma(\kappa)\Gamma(\lambda+1)}{\Gamma(\kappa+\lambda+1)},$$

hence

$$\int_0^1 f_{\kappa,\lambda}^{C}(T)\, dT = \int_0^C B_{\kappa,\lambda}(1-T/C)\, dT = C\frac{\Gamma(\kappa+\lambda)}{\Gamma(\kappa)\Gamma(\lambda)}\frac{\Gamma(\kappa)\Gamma(\lambda+1)}{\Gamma(\kappa+\lambda+1)} = \frac{\lambda}{\kappa+\lambda}C \tag{6}$$

A geometrical consequence of this property is that for any $0 \le C_1, C_2 \le 1$ the area ratio of the curves of $f_{\kappa,\lambda}^{C_1}$ and $f_{\kappa,\lambda}^{C_2}$ equals to the respected base ratio $C_1 / C_2$, i.e.

$$\frac{\int_0^1 f_{\kappa,\lambda}^{C_1}(T)\, dT}{\int_0^1 f_{\kappa,\lambda}^{C_2}(T)\, dT} = \frac{C_1}{C_2} \tag{7}$$

and:

$$I_{C_2}^{C_1} := \int_0^{C_2} f_{\kappa,\lambda}^{C_2}(T)\, dT - \int_0^{C_1} f_{\kappa,\lambda}^{C_1}(T)\, dT = \frac{\lambda}{\kappa+\lambda}(C_2 - C_1) \tag{8}$$

where $I_{C_2}^{C_1}$ is the area between $f_{\kappa,\lambda}^{C_1}$ and $f_{\kappa,\lambda}^{C_2}$ which is proportional to the base difference $C_1$ - $C_2$.

Figure 1 (right-side) shows the filled shapes of $f_{3,3}^{C}$ with $C$ ranging from 0 to 0.5 and step 0.1. Using the same household heuristics as before ($\kappa = \lambda = 3$) it results that the five areas between the curves are equal to 0.05[5].

[4] The integration region of the left hand side integral $0 \le t \le 1\text{-}T/C$ and $0 \le T \le C$ is the same with $0 \le T \le C(1\text{-}t)$ and $0 \le t \le 1$.

This property follows from the fact that $B_{\kappa,\lambda}(1-T/C)$ is a univariate polynomial expression of the composite variable *T* /*C*, which implies that if $0 \le T_1/C_1 = T_2/C_2 \le 1$, then $f^{C_1}_{\kappa,\lambda}(T_1) = f^{C_2}_{\kappa,\lambda}(T_2)$. Because $f^{C}_{\kappa,\lambda}$ is strictly increasing (and therefore 1-1) in the interval [0,*C*], it follows that if $f^{C_1}_{\kappa,\lambda}(T_1) = f^{C_2}_{\kappa,\lambda}(T_2)$, for some $0<C_1,C_2<1$, then

$$\frac{T_1}{C_1} = \frac{T_2}{C_2} \text{ or equivalently } \frac{T_1}{T_2} = \frac{C_1}{C_2} \qquad (9)$$

Equation 9 suggests that the difference required between transportation costs $T_1$ and $T_2$ to achieve equivalent transportation affordability can be determined by equating their ratio with the ratio of $C_1$, $C_2$. This difference can be adjusted by properly selecting the $\kappa$ and $\lambda$ parameters. If equality is maintained between the κ and λ, for any given value of the two the same cost increment will be required between successive $C_i$ in order to maintain the same levels of affordability. If different values are selected for $\kappa$ and $\lambda$ the horizontal separation of the family of curves will change, allowing for a diminishing effect as housing costs increase. It seems appropriate, however, to set up as a starting point a common value for $\kappa$ and $\lambda$ (i.e. $\kappa = \lambda = 3$) to apply across all $C_i$s.

It should be noted that both the shape and the horizontal separation of the functions can be calibrated by properly selecting the $\kappa$ and $\lambda$ parameters.

As regards the parameter *C*, this can be fixed by the policy-maker according to average household decision heuristics. It represents the maximum portion of household total expenditure devoted to transportation and housing together, that is considered affordable. A typical value of *C* is 0,5 which results from the following reasoning: traditionally housing is considered affordable by planners, lenders, and most consumers if it corresponds broadly to 30% of the monthly household income. Over that level of expenditure, any additional location or transportation cost will cause considerable reductions in other expenses, particularly food, clothing and entertainment. Transportation expenditures (excluding expenditures on luxury travel, such as long-distance vacation trips) can be considered unaffordable if they exceed 20% of a household's total expenditures *(1)*. By summing the two thresholds the value of 50% is derived for *C*.

It stems from the above that the model's 'agents', the households, monitor expenditure conditions including housing and transportation costs, and adjust their behavior accordingly. However, their rationality is bounded: In the tradition of Simon *(25)*, Morecroft *(26)* and Nelson et al. *(27)*, the households make decisions using routines and heuristics because the complexity of the decision environment exceeds their ability to optimize even with respect to the limited information available to them.

Here we draw on the literature cited above and the well-established tradition of bounded rationality and assume that households set affordability thresholds with intendedly rational decision heuristics. We have built the local rationality of households in the model in such a way so that the model generates different results when transportation options exist that allow transportation costs to be adjusted in a sufficiently quick manner relative to the dynamics of demand, such that the households' demand forecasts and estimates of their housing cost and spending capacity plans are reasonably accurate.

In this spirit, we model transportation affordability with realistic boundedly rational heuristics reflected by the parameter *C*. These are heuristics that allow us to capture different perspectives (trade-offs) for valuing affordability, including the "high spending capacity -

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[5] It is $I_0^{0.1} = I_{0.1}^{0.2} = I_{0.2}^{0.3} = I_{0.3}^{0.4} = I_{0.4}^{0.5} = \frac{3}{3+3}0.1 = 0.05$

leads to higher housing and transportation costs advantage - leads to increased affordability" logic which is not addressed by any other 'linear' methods (e.g. the H+T index).

Moreover, many studies show that forecasts are dominated by smoothing and extrapolation of recent trends *(28).* We capture such heuristics by assuming households extrapolate costs and expenditure on the assumption that recent trade-offs will continue. These dynamics are reflected by the $\kappa$ and $\lambda$ parameters.

*Final configuration of the Metric*

The Transportation Affordability Metric is completed by introducing the housing cost variable (*H*) in the $f^{C}_{\kappa,\lambda}$ (which satisfies property 1). *H* is defined as the portion of total expenditures devoted to housing and is ranging in the interval [0,*C*]. The results of this transformation are shown in Equation 10 below.

$$A^{C}_{\kappa,\lambda}(T,H) := f^{C-H}_{\kappa,\lambda}(T) := \begin{cases} B_{\kappa,\lambda}\big(1-T/(C-H)\big) & \text{if } 0 \le T+H \le C \\ 0 & \text{otherwise} \end{cases} \qquad (10)$$

It follows from Equation 10 that if $0 \le T_i \le C - H_i$, for $i$=1,2, then

$$\frac{T_1}{C-H_1} = \frac{T_2}{C-H_2} \text{ is equivalent to } A^{C}_{\kappa,\lambda}(T_1,H_1) = A^{C}_{\kappa,\lambda}(T_2,H_2) \qquad (11)$$

Equation 11 reflects the same calibrating condition discussed before in Equation 9; that is the required relation between transportation and housing costs to achieve equivalent transportation affordability. A schematical 2D and 3D representation of this relation is given in figure 2, assuming $C$ = 0.5 and $T$ / (0.5-$H$) = 0.2, 0.4, 0.6, 0.8, 1, respectively. The same figure also illustrates the overal variation of the Transportation Affordability Metric with respect to housing and transportation costs. The affordability values[6] that correspond to the *T, H* pairs satisfying these conditions are given in table 2 (second row).

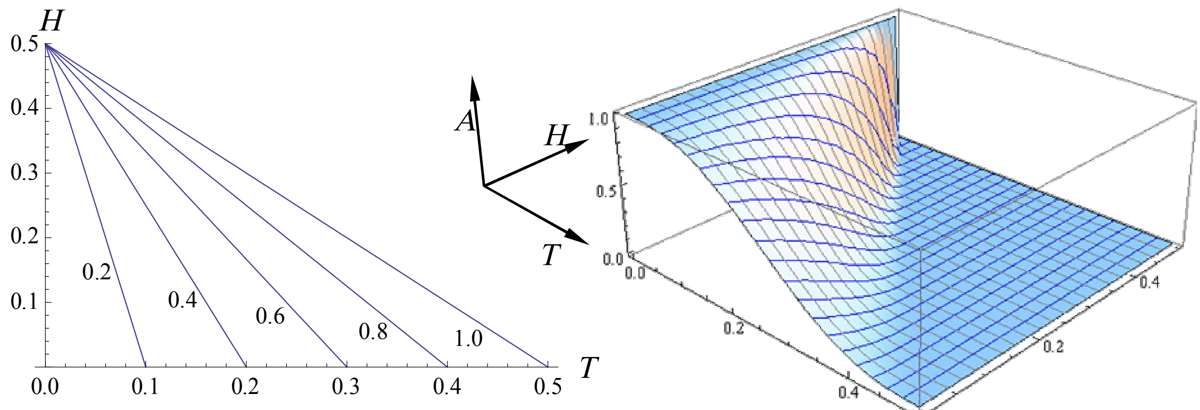


**FIGURE 2 Variation of TAM with respect to housing and transportation costs**

[6] Estimated as follows: $A^{0.5}_{3,3}(T,H) = P\big(T/(0.5-H)\big)$, where $P(x) := 1 - 10x^3 + 15x^4 - 6x^5$ (derived from Equation 5), and $x = \frac{T}{0.5-H}$

**TABLE 2 Relation between transportation, housing costs and affordability**

| $\frac{T}{0.5-H}$ | 0.2 | 0.4 | 0.6 | 0.8 | 1.0 |
|---|---|---|---|---|---|
| $A_{3,3}^{0,5}(T,H)$ | 0.942 | 0.682 | 0.317 | 0.057 | 0 |

## TESTING THE METRIC

The Transportation Affordability Metric was tested in the Samos region, an island at the Eastern Aegean, to demonstrate easiness to use while attempting to meet a series of efficiency criteria such as:

- The TAM is transparent
- Reduces the requirements in data collection
- Produces fairly reliable results when partial data is available
- Shows aspects not included in other methods.

A short survey was launched in the capital city of the island to assess the transportation and housing costs required for the analysis. Car ownership (*Co*), car use (*Cu*) and transit use (*Cp*) costs, including housing costs ($H_T$) and total average yearly expenditure of households (*E*) were evaluated based on the findings of the survey which included 400 interviews in a total population of about 10.000. The critical information used for the selection of the analysis zones included average household income, and transit availability within these zones. This example was not meant to be a full scale application of the transportation affordability analytical framework, but a demonstration of the TAM. Therefore, no transportation cost models were developed on Samos locational and household data. The car ownership, car use and transit use costs were derived directly from the survey, suggesting that the Metric performs well in cases of poor quality or higher-scale data. The results of the survey compiled for the computation of TAM are shown in table 3 below.

**TABLE 3 Transportation costs and household expenditure**

| Zone | *Co* (€) | *Cu* (€) | *Cp* (€) | $H_T$(€) | E(€) |
|---|---|---|---|---|---|
| Zone A | 2.236 | 1.613 | 62,40 | 5.760 | 33.860 |
| Zone B | 2.095 | 1.574 | 50,40 | 4.800 | 24.519 |
| Zone C | 1.830 | 1.852 | 42,00 | 4.200 | 23.352 |

Table 4 summarizes the results of the application example. It can be seen from the last column that zones B and C have similar transportation affordability which is due to tradeoffs that exist between the zones' average transportation and housing costs (note the signs: $T_2$ - $T_3$ = - 0.007777, and $H_2$ - $H_3$ = 0.015911).

**TABLE 4 Application example results**

| Zone | $T = \frac{Co+Cu+Cp}{E}$ | $H = H_T / E$ | $\frac{T}{0.5-H}$ | Affordability Metric |
|---|---|---|---|---|
| Zone A | 0.115517 | 0.170112 | 0.35017 | 0.764566 |
| Zone B | 0.151695 | 0.195767 | 0.498613 | 0.502602 |
| Zone C | 0.159472 | 0.179856 | 0.498127 | 0.503511 |

Figure 3 provides a schematic representation of the application results. It can be noted that zones B and C are laying on the same affordability line (red in 2D, black in 3D).

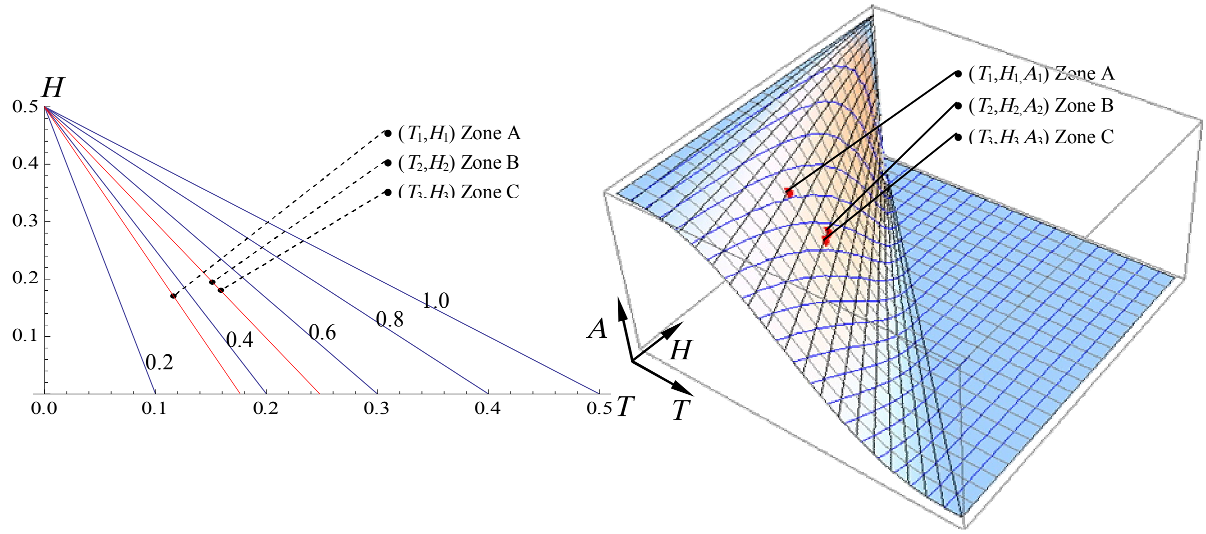


**FIGURE 3 Affordability values of tested travel zones**

## CONCLUSIONS AND RECOMMENDATIONS

Transportation affordability is an important economic and social issue. Unaffordable transport imposes significant financial burdens and reduces opportunities for disadvantaged people. Traditional planning hardly takes transportation affordability into account. Greater emphasis in this field would allow policy-makers to gain better understanding of transportation affordability and therefore design and promote more affordable transportation solutions.

There have been several attempts to measure transportation affordability. Most of them overlook the increased variation in transportation resources and costs across household groups and locational settings. To address this limitation, we have proposed a Metric for measuring transportation affordability based on the tradeoffs that households make between transportation and housing costs.

The following aspects of transport affordability were considered in building the TAM:

- Combined transport and housing costs (to account for possible tradeoffs)
- Transportation costs (including car ownership, car use and transit use costs, not just fuel or transit fares).
- Total expenditure (to avoid intrinsic weaknesses of household income).

The TAM is built on the beta cumulative density function, using data that is easily accessible in most organized countries. The Metric can be estimated at the neighborhood or higher zone level providing transport policy-makers with the information needed to make better planning decisions, which illuminate the implications of their policy and investment choices.

For the fine-tuning of TAM we used decision heuristics derived from the literature: transportation costs are to be considered affordable if they're under 20% of a household's total expenditures and the housing costs are at the range of 30 percent or less. For higher income households affordable accessibility allows virtually unlimited automobile travel, but for low-income households, it requires multi-modal transport systems with high quality public transport, taxi services, and also smart-growth cities, affordable housing in accessible locations well served by non-motorized modes and public transit.

Future work includes a full-scale application of the model in the wider London area using Holtzclaw's hypotheses to develop fitting models of households' transportation costs, namely car ownership, car use and transit use costs. To overcome some of the problems inherent in Holtzclaw's approach we will be using data on the policy-related variables of residential/job density and center proximity, and of the household characteristics coming from an agent-based micro-simulation model originally developed to assess the impacts of the Jubilee Line and the East London Line Extensions.

Future work aimed to improve the TAM will consider a variety of factors affecting affordability such as people's mobility needs, non-automotive transportation options and land use patterns. The challenge is to address a range of mobility needs; some people can easily satisfy their travel requirements with minimal cost, while others with limited physical ability or care giving responsibilities have to increase their transportation expenditure to do so. Transportation options also play an important role, especially public transit and non-automotive means which result in increased transportation affordability. Moreover, locational settings like density, land-use mix and street grid connectivity result in more nearby destinations, shorter trips and most affordable transportation. Areas of low residential density such as suburban and rural locations are likely to be more automobile-dependent, leading to decreased transport affordability.

## References


1. Litman T. Transportation Affordability - Evaluation and Improvement Strategies, *Victoria Transport Policy Institute*, www.vtpi.org, 2009
2. Serebrisky, T. Gomez-Lobo, A., Estupinan, N., Munoz-Raskin, R., Affordability and Subsidies in Public Urban Transport: What do We Mean, What Can be Done? *Transport Reviews, 29(6), pp. 715-739*, 2009.
3. Panou K., Kapros S., Lekakou M. and Syriopoulos T. Universal Service Obligations in Insular Areas, in *Proceedings (CD-ROM) of the 11th World Conference of Transport Research (WCTR)*, Berkeley, USA, 2007.
4. EEA - European Environmental Agency, Transport infrastructure investments (TERM 019) – Assessment, *EEA,* Jan 2011.
5. Wachs, M. Transportation Policy, Poverty, and Sustainability: History and Future. *Transportation Research Record: Journal of the Transportation Research Board,* 2010, volume 2163.
6. Litman T, Understanding Smart Growth Savings - What We Know About Public Infrastructure and Service Cost Savings, and How They are Misrepresented By Critics, *Victoria Transport Policy Institute,* Sep 10 2012.
7. Leigh, Scott & Cleary, Transit Needs & Benefits Study, *Colorado Department of Transportation,* www.dot.state.co.us, 1999
8. Litman T. Rail Transit In America: Comprehensive Evaluation of Benefits, *VTPI www.vtpi.org; at www.vtpi.org/railben.pdf,* 2004
9. Polzin S., Chu X. and Raman V.S. Exploration of a Shift in Household Transportation Spending from Vehicles to Public Transportation, *Center for Urban Transportation Research www.nctr.usf.edu; at http://www.nctr.usf.edu/pdf/77722.pdf,* 2008
10. FTA, Better Coordination of Transportation and Housing Programs to Promote Affordable Housing Near Transit, *Federal Transit Administration, USDOT and Department of Housing and Urban Development; at www.huduser.org/Publications/pdf/bettercoordination.pdf,* 2008
11. Kenworthy J. and Laube F. The Role of Light Rail in Urban Transport Systems: Winning Back Cities from the Automobile, presented to *The Fifth Light Rail Conference, UITP*, Melbourne Exhibition and Convention Centre, 8-11 October 2000.

12. Litman T. Transportation Cost and Benefit Analysis, *VTPI www.vtpi.org; at www.vtpi.org/tca*, 2005
13. CTOD and CNT. The Affordability Index: A New Tool for Measuring the True Affordability of a Housing Choice, *Center for Transit-Oriented Development and the Center for Neighborhood Technology, Brookings Institute www.brookings.edu; at www.brookings.edu/metro/umi/20060127affindex.pdf*, 2006.
14. Litman T. Smart Growth Reforms, *VTPI www.vtpi.org; at www.vtpi.org/smart_growth_reforms.pdf*, 2006
15. Miller E. Travel and Housing Costs in the Greater Toronto Area: 1986-1996, *Neptis Foundation www.neptis.org*, 2004
16. Makarewicz C., Haas P., Benedict A. and Bernstein S. Estimating Transportation Costs for Households by Characteristics of the Neighborhood and Household, *Transportation Research Board 87th Annual Meeting www.trb.org*, 2008
17. Arigoni D. Affordable Housing and Smart Growth: Making the Connections, Subgroup on Affordable Housing, *Smart Growth Network www.smartgrowth.org and National Neighborhood Coalition www.neighborhoodcoalition.org*, 2001
18. CNU, Parking Requirements and Affordable Housing, *Congress for the New Urbanism www.cnu.org; at www.cnu.org/node/2241*, 2008
19. EuroStat, 2005 Household Budget Survey, *EuroStat http://epp.eurostat.ec.europa.eu; at http://epp.eurostat.ec.europa.eu/pls/portal/docs/PAGE/PGP_PRD_CAT_PREREL/PGE_CAT_PREREL_YEAR_2008/PGE_CAT_PREREL_YEAR_2008_MONTH_06/3-19062008-ENAP.PDF*, 2008
20. BLS, 2005 Consumer Expenditures, *Bureau of Labor Statistics www.bls.gov; at www.bls.gov/cex/home.htm*, 2007
21. Holtzclaw J., Clear R., Dittmar H., Goldstein D. and Haas P. Location Efficiency: Neighborhood and Socioeconomic Characteristics Determine Auto Ownership and Use – Studies in Chicago, Los Angeles and San Francisco, *Transportation Planning and Technology*, 2002, Vol. 25, pp. 1–27
22. Andrews G.E., Askey R. and Roy R. Special Functions, *Cambridge University Press,* Cambridge UK, 1999
23. Press W.H., Flannery B.P., Teukolsky S.A. and Vetterling W.T. Gamma Function, Beta Function, Factorials, Binomial Coefficients and Incomplete Beta Function, *Numerical Recipes in FORTRAN: The Art of Scientific Computing,* Cambridge University Press, Cambridge UK, 2nd ed, 1992, pp. 206-209 and 219-223.
24. Abramowitz M. and Stegun I.A -Eds- Beta Function and Incomplete Beta Function, *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables,* 9th printing. New York: Dover, 1972, pp. 258 and 263.
25. Simon, H., Models of Bounded Rationality. *MIT Press,* Cambridge, MA, 1982.
26. Morecroft, J., Rationality in the analysis of behavioral simulation models. *Management Sci. 31(7) 900–916*, 1985.
27. Nelson, R., S. Winter, An Evolutionary Theory of Economic Change. *Belknap Press*, Cambridge, MA, 1982.
28. Sterman, J. D., Expectation formation in behavioral simulation models. *Behav. Sci. 32 190–211,* 1987.